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Generalised Radio Number of Odd Paths

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Abstract

Generalised Radio labelling of a graph *G,* an aspect signal assignment problem, and which generalises radio labelling problem, is a non-negative integer function on $V(G)$ of *G* to the set of positive integer, such that for $u,v \in V(G),\ |g(u)-g(v)| \geq diam(G)-d(u,v)+k,$ with $diam(G)$ being the diameter of *G* and *d*(*u,v*), the distance between vertices *u* and *v*. The optimum value of $span(g)$, which the difference between the largest and the smallest radio label on *V*(*G*) is the radio number $rng(G)$ of *G*. This work obtains the generalised radio number for the odd path P_n.

Keyword: Radio Labeling, Paths, Graphs

1. **Introduction**

The graph being considered in this work is simple, undirected graph G with vertex set *V(G)* and edge set is *E*(*G*). Member $e \in E(G)$ is defined as $e = uv$ where $u,v \in V(G)$ are incident members of *G*. For any $u,v \in V(G),\ d(u,v)$ is the positive integer distance between any u and v while the diameter *diam*(*G*) of *G* is the longest path between in *G*.

Introduced by Hale (Hale, 1980), radio number problem solves some frequency assignment and signal interference problems. For *f*, a non-negative function on *V(G)*, the radio labelling problem satisfies $|f(u)-f(v)| \geq diam(G)-d(u,v)+1$ for $u,v \in V(G)$. Where $f_{\min}$ is the least label on G and $f_{\max}$, the largest label. Based on this, $span(f) = f_{\max} - f_{\min}$, which when is optimum, is the radio number $rn(G)$ of G. The radio labelling condition guarantees that each member of $V(G)$ has a unique radio label, which yields the fact that $rn(G) \geq |V(G)|$. Radio numbers of some graph have been obtained despite the difficult work involved in obtaining this numbers. For simpler graphs such as a path P_n with $V(P_n) = n$, $n > 2$, Liu and Zhu (Liu & Zhu , 2005), with $rn(P_n) = 2r(r-1)+1,$ for $n = 2r$ and $rn(P_n) = 2r^2+2,$ for $n = 2r + 1,$ improved the bounds obtained in Chatrand, et. al. 2001, 2005. Radio numbers have also been derived for star graph and other sunlets graphs (Rani & Parvathi, 2016) and a few product graphs. Jiang, (Jiang, 2014), obtained, among other results the radio number for a grid graph $G_{mn}= P_m \square P_n$, the Cartesian product of paths P_m and P_n , where m is even and n is odd as

$$rn(G_{mn}) = \frac{mn^2 + nm^2 - n}{2} - mn - m + 2 .$$

While bounds of radio numbers were obtained for stacked-book graphs $H_{m,n} = S_m \square P_n$ by Ajayi & Adefokun, (2018) before they precisely determined numbers in 2019 (Adefokun & Ajayi, 2019). the radio number of edge joint graphs and other were obtained by Naseem et. al. (2018). Adefokun (2019) introduces generalised radio number problem (Adefokun, 2019) as the positive integer function g on $u, v \in V(G)$ such that $|g(u)-g(v)| \geq diam(G)+d(u,v)+k,$ where $k \geq 1$ is a positive integer in other to guaranty a more stabilised network with no chance of interference, since the large the value of $k,$ the wider the frequencies are in the network. Generalised radio number $rng(G)$ of graph G was obtained for the path P_n where $n \geq 4$ is even (Adefokun , 2019). This work, adopting a different technique, look to derive the radio number for odd path $P_n, n \geq 3$.

2. Preliminaries

Path P_n consists of $n-$ even vertices even $n-1$ edges. This clearly demonstrates that $diam(P_n) = n-1.$

3. Results

Lemma 1. Let p_n be a path of order n where n is odd and let $G(*) \subseteq P_n$ be a subgraph of P_n . induced by vertices $v_1, v_{\frac{n+1}{2}} \subset V(P_n).$ Then, $rng(G(*)) = \frac{n-1}{2} + k$ in P_n .

Proof. Let $g(v_{\frac{n+1}{2}}) = 0.$ Clearly, $d(v_1, v_{\frac{n+1}{2}}) = \frac{n-1}{2}.$ From the definition of labelling function g , $g(v_1) \geq g(v_{\frac{n+1}{2}}) + diam(P_n) - d(v_1 - v_{\frac{n+1}{2}}) + k$. Thus, $g(v_1) \geq 0 + n - 1 - (\frac{n-1}{2}) + k$ and so, $g(v_1) \geq \frac{n-1}{2} + k.$ Since $|V(G(*))| = 2,$ then $g(v_1) = \frac{n-1}{2} + k.$ ■

Next, we define another subgraph of P_n .

Definition 1. Let P_n be an odd path. Then $G(**)$ is a subgraph of P_n defined as $G(**) = P_n \setminus \{G(*) + v_n\}.$

Remark 1. Clearly, $V(G(**)) = \{v_2, v_3, \cdots, v_{\frac{n-1}{2}}, v_{\frac{n+3}{2}}, \cdots, v_{n-1}\}$. Now, $G(**)$ is induced by $V(G(**))$. Next we obtain a lower bound for the generalised radio number $rng(G(**))$ of $G(**)$ in P_n. To achieve this, we define another a subgraph of $G(**)$.

Definition 2. Let $G_r^{'} \subseteq G(**)$ be a subgraph of $G(**)$ such that $G_r^{'}$ is induced by $v_r, v_{r+\frac{n-1}{2}} \in V(P_n)$.

It should be observed that P_n contains $\frac{n-3}{2}$ copies of $G_r^{'}$ subgraphs. Next, derive the radio number of $G_r^{'}$.

Lemma 2. For $G_r^{'} \subseteq G(**)$, then, $rng(G_r^{'}) = g(v_r) + \frac{n-1}{2} + k$.

Proof. The claim follows directly from definitions.

Remark 2. It can be noted that $span(g)$ of g in $G_r^{'}$ is $\frac{n-1}{2} + k$. Next, we obtain the lower bound for the radio number of $G(**)$. $\frac{n-1}{2} + k$.

Lemma 2. For $G(**) \subset P_n$, then, $rng(G(**)) \geq \frac{1}{2}[(n-3)^2] + (n-4)k$.

Proof. Without loss of generality, we set $r = \frac{n-1}{2}$ Now, $G^{'}_{\frac{n-1}{2}}$ is induced by $v_{\frac{n-1}{2}}, v_{n-1}$. So, suppose that $0 = g_{\min}(G(**)) = g(v_{\frac{n-1}{2}})$, then, $g(n-1) \geq 0 + span(g) = \frac{n-1}{2} + k$. Now, $d(v_{n-1}, v_{\frac{n-3}{2}}) = \frac{n+1}{2}$. Therefore,

$$g(v_{\frac{n-3}{2}}) \geq g(v_{n-1} + \tfrac{n-3}{2}) = \tfrac{n+1}{2}.$$

$$\geq \tfrac{n-1}{2} + \tfrac{n-3}{2} + 2k \ .$$

In the end, with some careful iteration, it can be obtained that

$$g_{\max}(**) = g(v_{\frac{n+3}{2}}) \geq \tfrac{(n-3)}{2}\tfrac{(n-1)}{2} + \tfrac{(n-5)}{2}\tfrac{(n-3)}{2} + (n-4)k$$

$$\geq \tfrac{1}{2}[(n-3)^2] + (n-4)k.$$

Now, we can proceed to obtain a lower bound for the generalised radio number of P_n , $n-$ odd.

Theorem 1. For P_n, $n-$ odd, $rng(P_n) \geq \frac{1}{2}(n^2 - 4n + 9) + (n-2)k$.

Proof. It is stated earlier that $P_n = G(*) \bigcup G(**)$. Now, for $v_1 \in V(G(**))$, $d(v_1, v_{\frac{n-1}{2}}) = \frac{n-3}{2}$. Now, from an earlier lemma, $g(v_1) \geq \frac{n-1}{2} + k$. Thus,

$$g(v_{\frac{n-1}{2}}) \geq g(v_1) + n - 1 - (\tfrac{n-3}{2}) + k$$

$$\geq \tfrac{n-1}{2} + k + n - 1 - (\tfrac{n-3}{3}) + k$$

$$\geq \tfrac{n-1}{2} + \tfrac{n+1}{2} + 2k$$

$$\geq n + 2k.$$

Now, if we set $g(v_{\frac{n-1}{2}}) \geq n + 2k$ as the new $g_{\min}(g(**))$, then, $g(v_{\frac{n+3}{2}})$, the new $g_{\max}(G(**))$ becomes

$$\begin{aligned} g_{\max}(P_n) &\geq g_{\min}(G(**)) + \tfrac{1}{2}[(n-3)^2] + (n-4)k \\ &\geq n + 2k + \tfrac{1}{2}[(n-3)^2] + (n-4)k \\ &\geq \tfrac{1}{2}(n^2 - 4n + 9) + (n-2)k. \end{aligned}$$

Remark 3. If we set $k = 1$, then the lower bound of $rng(P_n)$ has the same value as $rn(P_n)$ as obtained in Liu & Zhu , 2005.

Having obtained the lower bound of the generalised radio number of P_n we now proceed to establish the upper bound of $rn(P_n)$.

Theorem 2. For P_n , $n-$ odd, $rng(P_n) \leq \frac{1}{2}[(n^2 - 4n + 9)] + (n-2)k.$

Proof. Let $g(v_{\frac{n+1}{2}}) = 0.$ Since $d(v_{\frac{n+1}{2}}, v_1) = \frac{n-1}{2}$, then, either $g(v_1)$ and $g(v_n)$ can be labelled next. Without loss of generality, let

$$\begin{aligned} g(v_n) &= g(v_{\frac{n+1}{2}}) + n - 1 - (\tfrac{n-1}{2}) + k \\ &= \tfrac{n-1}{2} + k. \end{aligned}$$

Clearly, $d(v_1, v_n) = diam(P_n) = n - 1.$ Then $g(v_1) = \frac{n-1}{2} + 2k.$ Now, $d(v_n, v_{\frac{n+3}{2}}) = \frac{n-3}{2}.$ So,

$$\begin{aligned} g(v_{\frac{n+3}{2}}) &= \tfrac{n-1}{2} + k + \tfrac{n+1}{2} + k \\ &= n + 2k. \end{aligned}$$

This label is appropriate with respect to $g(v_1)$. Now, $d(v_{\frac{n+3}{2}}, v_2) = \frac{n-1}{2}.$ Thus.

$g(v_2) = n + 2k + \frac{n-1}{2} + k = \frac{3n-1}{2} + 3k.$ The distance $d(v_2, v_{\frac{n+5}{2}}) = \frac{n+1}{2}.$ Clearly

$$\begin{aligned} g(v_{\frac{n+5}{2}}) &= \tfrac{3n-1}{2} + 3k + \tfrac{n-3}{2} + k \\ &= 2(n-1) + 4k. \end{aligned}$$

Likewise $d(v_{\frac{n+5}{2}}, v_3) = \frac{n-1}{2}$, and thus,

$$\begin{aligned} g(v_3) &= 2(n-1) + 4k + \tfrac{n-1}{2} + k \\ &= \tfrac{5(n-1)}{2} + 5k. \end{aligned}$$

Now, $d(v_3, v_{\frac{n+7}{2}}) = \frac{n+1}{2}.$ Thus, $g(v_{\frac{n+7}{2}}) = \frac{5(n-1)}{2} + 5k + \frac{n-3}{2} + k = 3n - 4 + 6k.$ Likewise again, $d(v_{\frac{n+7}{2}}, v_4) = \frac{n-1}{2}$ and therefore,

$$\begin{aligned} g(v_4) &= 3n - 4 + 6k + \tfrac{n-1}{2} + k \\ &= 3n - 4 + \tfrac{n-1}{2} + 7k \\ &= \tfrac{7n-9}{2} + 7k. \end{aligned}$$

Based on the scheme above, $g(v_{\frac{n-1}{2}}) = g_{\max}(P_n),$ and iteratively,

$$\begin{aligned} f(v_{\frac{n-1}{2}}) &= \tfrac{1}{2}[(n-2)n - (2n-9)] + (n-2)k \\ &= \tfrac{1}{2}[n^2 - 4n + 9] + (n-2)k. \end{aligned}$$

Theorem 3. For path P_n , $n-$ odd, $rng(P_n) = \frac{1}{2}(n^2 - 4n + 9) + (n-2)k.$

Conclusion. The result obtained in this work, apart from presenting a generalisation of radio number of odd paths, it also present an alternative proof for the radio number of paths. By setting $k = 1,$ we have the result in Liu & Zhu , 2005

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