CRAWFORD UNIVERSITY
FAITH CITY, IGBESA, OGUN STATE
College of Natural and Applied Sciences
Harmattan Examination, (2008/2009 Session)
MTH307 (Complex Analysis I)

Answer 4 questions in all, Time allowed: 2hrs

1.(a) Define the following terms
(i) Neighbourhood of a point (ii) Limit points (iii) Interior points
(iv) Boundary points (v) Closed points (vi) Open set (vii) Domain
(viii) Continuity of a function

(b) Show that $|z_1 - z_2|^2 + |z_1 + z_2|^2 = 2|z_1|^2 + 2|z_2|^2$ and deduce that

$$|\alpha + \sqrt{\alpha^2 - \beta^2}| + |\alpha - \sqrt{\alpha^2 - \beta^2}| = |\alpha + \beta| + |\alpha - \beta|$$ all the numbers concerned being complex.

2. (a) State the necessary and sufficient conditions for a function $f(z) = u(x,y) + iv(x,y)$ to be analytic in a region R. Hence show that the function $f(z) = e^x(\cos x + i \sin x)$ *is no where analytic*

(b) Show that $f(z) = 2x(1-y) + i(2y + x^2 - y^2)$ is analytic

3. (a) State the Laurent's theorem. Expand $f(z) = \dfrac{1}{(z+2)(z+5)}$ valid for

$2 \prec |z| \prec 5$

(b) Evaluate $\displaystyle\int_0^{\infty} \frac{dx}{x^6 + 1}$

4. (a) Find the roots of (i) $(-1+i)^{1/3}$ *and* (ii) $(-2\sqrt{3} - 2i)^{1/4}$ and locate them graphically.

(b) Explain the following terms
(i) Conformal mapping (ii) Removable singularity (iii) Bilinear transformation
(iv) Branch points.

5. (a) Find Laurent series about the indicated singularity for each of the following functions. Name the singularity in each case and give the region of convergence of each series:

(i) $\dfrac{e^{2z}}{(z-1)^3}$ (ii) $(z-3)\sin\dfrac{1}{z+1}$ (iii) $\dfrac{z}{(z+1)(z+2)}$

(b) Enumerate ten importance of complex analysis to your field of study.

6. State and prove the Taylor's theorem.