CRAWFORD UNIVERSITYFAITH CITY, IGBESA

SCHOOL OF PART-TIME STUDIES DEPARTMENT OF COMPUTER SCIENCES 2021/2022 HARMATTAN

SEMESTER EXAMINATION

COURSE TITLE: Discrete Mathematics **COURSE CODE**: CSC 412

LEVEL: 400

INSTRUCTION: ANSWER ANY FOUR QUESTIONS **TIME ALLOWED**: 3 Hours

Question 1

a. Show for any integer that $\sqrt{2}$ is irrational using prove by contradiction
b. Prove by contradiction that for all nonnegative integers $a > b$ the difference of squares $a^2 - b^2$ does not give remainder 2 on divided by 4
c. Prove by mathematical induction that for all nonnegative integers n

$$0 + 1 + 3 + 4 + \ldots + n = \frac{n(n+1)}{2}$$

Question 2

a. Prove by course-of-value induction that for every integer $n \geq 2$ can be written as a product of prime numbers

b. Prove that for every integer $n \geq 1$ can be written uniquely as a product of prime numbers in ascending order, i.e. as a product

$$P_1{}^{r_1} - P_k{}^{r_k}$$

Where $P_1, \ldots, P_k$ are primes such that $P_1 < \ldots < P_k$ and $r_1, \ldots, r_k$ are positive integers.

c. Explain the following term

i. Set.

ii. The Boolean identities for sets (at most four)

Question 3

a. Let A and B be sets, prove that

$$A \subseteq B \Leftrightarrow A \cap B = A$$

b. Explain the Truth table

c. Prove that a function $f: X \to Y$ is subjective iff it has an inverse function

Question 4

a. Define the following;

i. A directed graph
ii. An Equivalence relation

b. Let R be an equivalence relation on a set X. The set X/R = def$\{\{x\}_R / x \in X\}$ of equivalence classes with respect to R is a partition of the set X. Moreover, $\{x\}_R = \{y\}_R$ iff xRy for all$x, y \in X$.

C. Explain the following;

a. A partial Order
b. A Total order

Question 5

a. Prove that any subset of natural numbers is countable.
b. Prove that any subset of natural numbers is countable.

c. Prove that a Set B is countable iff there is an injection $f: B \to A$ into a set A which is countable.

Question 6

a. Prove that the set $\mathbb{N}\ x\ \mathbb{N}$ is countable.

b. Prove that the set of real numbers $\mathbb{R}$ is uncountable